Calculus Warm Up #4-2

Use implicit differentiation to find the derivative of:

1. $7y^4 - 10x^2 = 3y^2$
2. $-4x^3 + 2xy^2 - 2y = 5$

HW Questions: p. 141

In Exercises 1–16, find dy/dx by implicit differentiation and evaluate the derivative at the indicated point.

Equation	*Point*
1. $x^2 + y^2 = 16$	$(3, \sqrt{7})$
3. $xy = 4$	$(-4, -1)$
5. $x^{1/2} + y^{1/2} = 9$	$(16, 25)$
7. $x^3 - xy + y^2 = 4$	$(0, -2)$

9. $y^2 = \dfrac{x^2 - 9}{x^2 + 9}$ (3, 0)

11. $(x^3)(y^3) - y = x$ (0, 0)

product rule

13. $x^{2/3} + y^{2/3} = 5$ (8, 1)

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

15. $x^3 - (2x^2)(y) + (\overset{f}{3x})(\overset{g}{y^2}) = 38$ (2, 3)

$$f\,g' + f'g$$

$$3x^2 - \left(2x^2\frac{dy}{dx} + 4xy\right) + \left(3x \cdot 2y\frac{dy}{dx} + 3y^2\right) = 0$$

$$-2x^2\frac{dy}{dx} + 6xy\frac{dy}{dx} = -3x^2 + 4xy - 3y^2$$

$$\frac{dy}{dx} = \frac{-3x^2 + 4xy - 3y^2}{-2x^2 + 6xy}$$

Now plug in (2,3)

In Exercises 17–20, find the slope of the tangent line to the graph at the indicated point.

17. Witch of Agnesi:
$(x^2 + 4)y = 8$
Point: (2, 1)

$2xy + (x^2+4)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2xy}{x^2+4}$

@ $(2,1) \rightarrow \frac{-2(2)(1)}{2^2+4}$

$= -\frac{1}{2}$

In Exercises 25–30, find d^2y/dx^2 in terms of x and y.

25. $x^2 + xy = 5 \longrightarrow$ $\frac{x\cancel{y}}{\cancel{x}} = \frac{5-x^2}{x}$

$y = \frac{5}{x} - x$

$y' = -5x^{-2} - 1$

$y'' = 10x^{-3}$

$y'' = \frac{10}{x^3}$

27. $x^2 - y^2 = 16$

$2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{\cancel{2}x}{\cancel{2}y}$

$\frac{dy}{dx} = \frac{x}{y} \rightarrow xy^{-1}$

$\frac{d^2y}{dx^2} = x(-1)y^{-2}\frac{dy}{dx} + (1)y^{-1}$

$\frac{d^2y}{dx^2} = -\frac{x}{y^2}\cdot\frac{x}{y} + \frac{1}{y}\cdot\frac{y^2}{y^2}$

$\frac{d^2y}{dx^2} = \frac{-x^2+y^2}{y^3} \longleftarrow$ $x^2-y^2=16$ so factor out -1 & replace numerator

In Exercises 25–30, find d^2y/dx^2 in terms of x and y.

29. $\boxed{y^2 = x^3}$

$2y\frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{2y}$

$\frac{d^2y}{dx^2} = \frac{\left[2y(6x) - 3x^2 \cdot 2\frac{dy}{dx}\right]}{4y^2}$

$= \frac{1}{4y^2}\left[\frac{y}{y}\frac{(12xy)}{1} - \frac{\cancel{6}^{3}x^2}{1} \cdot \frac{3x^2}{\cancel{2}y}\right]$

$= \frac{1}{4y^2}\left[\frac{12xy^2 - 9x^4}{y}\right]$ $\qquad y^2 = x^3$

$= \frac{12x^4 - 9x^4}{4y^3}$

$= \frac{3x^4}{4y^2 \cdot y}$

$= \frac{3x^4}{4x^3y}$

$\frac{d^2y}{dx^2} = \frac{3x}{4y}$

In Exercises 31 and 32, find equations for the tangent line and normal line to the given circle at the indicated points. (The **normal line** at a point is perpendicular to the tangent line at the point.)

so slopes are opposite & reciprocal

31. $x^2 + y^2 = 25$, $(4, 3)$ and $(-3, 4)$

$\frac{dy}{dx} = 0$

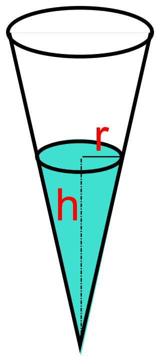
In Exercises 33 and 34, find the points at which the graph of the given equation has a vertical or horizontal tangent line.

undef. so set denom = 0

33. $25x^2 + 16y^2 + 200x - 160y + 400 = 0$

3.7 Related Rates

Today: Finding rates of change of two or more related variables that are changing with respect to time.

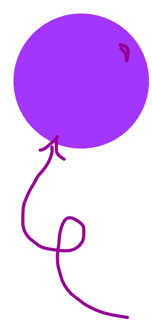


$$V = \frac{1}{3}\pi r^2 h$$

As water fills this cone at a steady rate, the volume of water changes over time. The radius and height of the water also change over time.

All 3 rates are changing with respect to time.

Another Example: As you blow up a balloon, the radius of the balloon is increasing and so is the volume of air inside.



Are the rates of change constant? Is the radius of the balloon and the volume of air changing at the same rate?

Volume of a sphere:

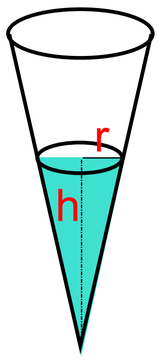
$$V = \frac{4}{3}\pi r^3$$

Differentiate with respect to time to find the relationship between the variables and their rates of change.

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$$

Related Rates equation $\rightarrow$ $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

Implicit Differentiation with respect to time will give us the relationship between the rates.



$$V = \left(\frac{1}{3}\pi r^2\right)(h)$$

Product Rule!

$$\frac{dV}{dt} = \frac{1}{3}\pi r^2 (1)\frac{dh}{dt} + \frac{1}{3}\pi \cdot 2r\frac{dr}{dt} \cdot h$$

$$\frac{dV}{dt} = \frac{\pi}{3}\left[r^2\frac{dh}{dt} + 2rh\frac{dr}{dt}\right]$$

Process for solving related rate problems:

1. Assign variables for all known and unknown quantities. Make a sketch and label it if possible.

2. Write equations for the relationship(s) between the variables.

3. Differentiate with respect to time, t, to come up with the related rates equation.

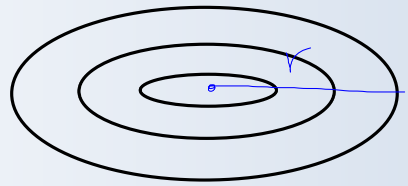
4. Plug in what you know and solve for the required rate of change.

A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius of the outer ripple is increasing at a constant rate of 1 foot per second. When this radius is 4 feet, at what rate will the total area of the disturbed water be increasing?

$\frac{dr}{dt} = 1 \frac{ft}{sec.}$

when $r = 4$

find $\frac{dA}{dt}$



$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(4)(1)$

$\frac{dA}{dt} = 8\pi$

$\approx 25.1 \ ft^2/sec$

x and y are both differentiable functions of t and are related by the equation: $y = x^2 + 3$

Differentiate with respect to t.

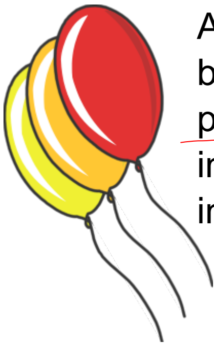
Given that $\frac{dy}{dt} = 2$, $\boxed{\text{find } \frac{dx}{dt}}$ when x = 3.

$\frac{dy}{dt} = 2x \frac{dx}{dt}$

$2 = 2(3) \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{1}{3}$

Another to try!



Air is being pumped into a spherical balloon at a rate of 4.5 cubic inches per minute. Find the rate of change in the radius when the radius is 2 inches.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \boxed{\frac{dr}{dt}}$$

$$4.5 = 4\pi(2)^2 \frac{dr}{dt}$$

$$\frac{4.5}{16\pi} = \frac{dr}{dt}$$

$$\frac{dr}{dt} \approx 0.884 \text{ in/min}$$

HW: p. 148 # 1 - 9,

13 - 17 odd

Chapter 3 Test: Next Monday.